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REJOINDER—THE PREDICTION OF THE TRANSPORT PROPERTIES OF A TURBULENT FLUID

1. INTRODUCTION

THE PURPOSE of this note is to reply to certain criticisms of our paper [1] made above by V. Walker and C. J. Lawn. The former also quotes a comment by P. Bradshaw regarding another test of our theory and implies that it fails this test. We welcome these discussions of our paper and appreciate some of the points made and reply to others in detail below. However, we take some exception to Walker's remark that "the usefulness of the theory proposed must rest entirely on the correctness of its predictions, and rather more evidence is required than diffusivity ratio comparisons". He expresses this as being *his* view—and this is so basic that it is discourteous to suggest that our view might differ. In fact, the following extract from a paper [2] of one of us presented at the Osborne Reynolds Centenary Symposium, Manchester, 1968, shows that we have probably a better appreciation of the limitations of our theory and of the need for further development and checking than has Walker. "It is evident that these results are reasonably satisfactory but this does not in itself imply that our ideas will be successful when applied to determine absolute values of heat, mass, and momentum transfer. However, we are in the process of trying to develop them towards these ends. It may be that we shall eventually have to bring in additional ideas beyond those we at present intend. But we feel that we must first explore fully the potential inherent in the ideas which arise naturally in the entity model". However, in addition to our awareness of the present limitations of the theory we also have some recognition of its potential.

2. THE POINTS RAISED BY WALKER

The apparently serious objection raised by Walker to the theory in our paper [1] is his statement that our expression for eddy diffusivity for momentum transport depends on the molecular viscosity. Such a result would be in conflict with the well known viscosity independence of flow in the logarithmic region of turbulent flows. But in fact Walker's statement is unjustified. It is clear from equation (22) that in our theory ε_μ is proportional to $\mu<\psi N_R^2>$ but nothing in the theory indicates that $<\psi N_R^2>$ is expected to be independent of viscosity. We cannot understand why Walker has made this assumption. The obvious appropriate procedure is to *deduce*, from the known independence of ε_μ on μ that in the logarithmic region of turbulence $<\psi N_R^2>$ varies inversely as μ. We indeed hope to extend our theoretical argument to investigate whether this universal variation can be established theoretically, and this would completely cover the point. Meanwhile separate theoretical confirmation may be adduced from the work of Leslie [3] who has used a wavenumber analysis to expand the velocity field in a series of spherical harmonics or symmetry classifications and then used a Focker–Planck or exponential time dependence for the infinitesimal inertial response. The form of his result includes a group corresponding to our $\mu < \psi N_R^2 >$, and the numerical multipliers differ only by four per cent. We feel therefore that the particular objection raised by Walker cannot be accepted.

In regard to another of Walker's points. It is true that for the limited objectives of diffusivity ratios tackled in the reference paper the size, shape, and initial velocities of the entities vanish, but this is no criticism. These are the conceptual elements of the theory and must be established and developed. They do *not* cancel out when other matters are investigated, and we are fully aware that more stringent tests of the theory will arise in such cases. One of us has already made this point (2). However, in such tests as have been made so far, where these particular factors have not cancelled, we have still found reasonable confirmation. In free turbulence, for example, the diffusivity ratios are found to be dependent on the shape parameter in particular. This predicted difference from the independence in pipe or channel flow is itself an important result.

The alternative derivation of the probability distribution for (λ/λ^*) developed by Walker is interesting and helpful, although it must be stressed that our theory is not dependent on whether one adopts the picture of entities surrounded by entities or by quiescent fluid. The linearity of equations (3) and (34) ensures that providing the correlation between the properties of the entity and its immediate surroundings is not strong then the average behaviour is represented correctly by equations (4) and (36) regardless of the structure of the surrounding fluid. The case of remote entities surrounded by quiescent fluid is a special case of the theory obtained by allowing some of the entities to have $v_0 = 0$ and the expectation operators take account of this when the diffusivities are developed.

The suggestion that the analysis only considers the contribution to the transport processes from entities traversing the plane $y = 0$ in one direction is incorrect. It is made clear prior to equation (17) that the expectation operators take account of all entities which traverse the plane $y = 0$. In determining the expectation value of the function of (λ/λ^*) in equation (20), we take the range of values of (λ/λ^*), a positive quantity for all entities, to be $0 < (\lambda/\lambda^*) < 1{\cdot}0$. The object in retaining the ratio (λ/λ^*) in the analysis is that it is a positive quantity for entities travelling in either direction. We have nowhere inferred that λ is taken to be positive.

Walker is correct in pointing out that there is an inconsistency between equations (40), (41) and (42). We regret an error in omitting the specific heat C_p which should appear as a multiplier on the right hand side of the equations. This omission, is not carried through and in the subsequent equation C_p has been recovered. It is of no great consequence to the theory whether ε_μ and ε_H are described as diffusivities or turbulent viscosity and conductivity.

The analysis for turbulent energy transport is similar to that of momentum or thermal energy transport once the appropriate decay equation, in this case equation (24), has been determined. The point which perhaps needs emphasizing is that when the entity is in the terminal stages of its trajectory, it does not adjust to the energy density of its surroundings. Decay of turbulent energy during the entity motion is described by equation (24) and is not dependent on any interaction law involving the difference in energy density between the entity and its surroundings.

Finally it must be pointed out that the analysis given in the paper is in many cases similar to that made in a Fourier analysis of the turbulent field and the "assumptions" made are not as unreasonable as might be implied by Walker's final remarks. Later developments of the theory have produced results which compare most favourably with those obtained by much more complex analyses in wavenumber space and furthermore applications of the theory to relaxation phenomena has penetrated beyond any other theoretical method and with credible results.

3. THE POINTS RAISED BY LAWN

It is clear from the comments made by Lawn that the overall train of thought in the paper has been misunderstood. The reasoning behind our development has been discussed by Silver [2] and is closely related to the information on turbulent flows obtained by a wavenumber analysis of the field. Such an analysis, as given for example by Batchelor [4] shows that at low flow Reynolds numbers the interaction between eddies is largely viscous and particularly so for the smaller size of eddies. As the Reynolds number increases, however, the rôle of inertial interaction, or interactions caused by small scale motion, becomes increasingly important. At the same time the distribution of energy amongst the eddy scales changes and as the Reynolds number increases the contribution of the larger eddies to the energy density and shear stress increases. Thus in our analysis we have first of all considered the problem of molecular interaction between entities and in equations (8) and (9) have established the correspondence between entity scale and the scale δ of the dissipation eddies which are known to have interactions of a purely molecular form. Subsequently we have established the transport properties of a fluid composed of such entities which must, therefore, correspond to a low Reynolds number flow regime where the contribution to the transport processes from the larger scales is not important. Hence this part of the analysis represents one extreme of turbulent motion.

At much higher Reynolds number the transport processes are, as Lawn correctly points out, dominated by eddies of a scale much greater than the dissipation scale, but it is well known that the interactions between these eddies and their surroundings is largely inertial and molecular interactions become unimportant. A consequence of this is that the prediction made by Lawn of the Reynolds number of the "energy containing entity" is in no way related to the subsequent motion or life path of the entity since the Reynolds number is based on *molecular* viscosity whereas the entity motion is determined by *inertial* interaction. As a result the objections made by Lawn to our theory are all based on a fallacy and need not be discussed in detail. It is worth pointing out, however, that his estimate of the unbounded life path of an "energy containing entity" is not compatible with observations of the mixing region of a turbulent jet at outlet from a pipe or channel.

In our theory we make a simple approximation for the inertial interaction by assuming that this interaction can be approximated by a gradient diffusion process with a diffusivity appropriate to the smaller scale or dissipation entity system. This approximation has a sound physical basis and is similar to that of Heisenberg [5]. It becomes clear now that since this diffusivity of the small scale entity system is much greater than the corresponding molecular diffusivity the appropriate Reynolds number for the larger entities will be much less than that used by Lawn and in this way λ^* assumes reasonable values and the inter-entity interaction we have assumed is not in any way compromised by the experimental results of Goldstein [6], Froessling [7] or Comte-Bellot [8].

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